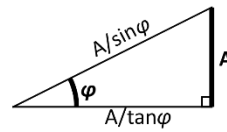
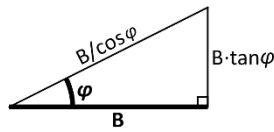
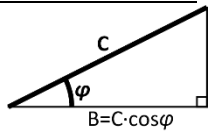


Formelsamling til TEK-1011 Anvendt mekanikk

Trigonometri

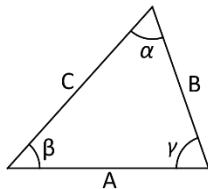
Rettvinklet trekant



Pythagoras:

$$A^2 + B^2 = C^2$$

Generell trekant



Sinussetningen:

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

Cosinussetningen:

$$A^2 = B^2 + C^2 - 2BC \cos \alpha$$

Geometri (se også tabell på siste side)

Volumsenter (tilsv. for y_c og z_c)

$$x_c = \frac{\sum(V_i x_i)}{\sum V_i}$$

Arealsenter (tilsv. for z_c)

$$y_c = \frac{\sum(A_i y_i)}{\sum A_i} = \frac{S_z}{A}$$

Arealmoment

$$S_z = \int_A y dA = \sum(A_i y_i)$$

Annet arealmoment

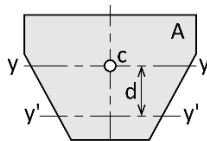
$$I_y = \int_A z^2 dA$$

Tverrsnittsmodul:

$$W_y = \frac{I_y}{|z_{maks}|}$$

Steiners formel

$$I_{y'} = I_y + d^2 A$$



$I_{y'}$ = Annet arealmoment om y' -akse

I_y = Annet arealmoment om y -aksen gjennom arealsenter

d = Avstand mellom y -akse og y' -akse

A = Flatens areal

Annet arealmoment når areal legges sammen

$$I_{1+2} = I_1 + I_2 + d^2 \frac{A_1 A_2}{A_1 + A_2}$$

Annet arealmoment når areal trekkes fra

$$I_{1-2} = I_1 - I_2 - d^2 \frac{A_1 A_2}{A_1 - A_2}$$

Momentsetningen

Resultantens moment M_R om fritt valgt momentpunkt er lik summen av kraftsystemets momenter om punktet.

$$M_R = \sum(F_i a_i) + \sum M_i$$

$$M_R = R \cdot a_R \text{ så lenge resultanten ikke blir et kraftpar}$$

Tyngde og tyngdepunkt

Tyngde: $G = g \cdot m = g\rho V$ hvor $g = 9,81 \frac{\text{N}}{\text{kg}}$

Tyngdepunkt: $x_c = \frac{\sum G_i x_i}{\sum G_i} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum(\rho_i V_i x_i)}{\sum(\rho_i V_i)}$

Beliggenhet ift fritt valgt referansepunkt (tilsv for y_c og z_c)

Resultant

Komponenter

$$R_x = \sum(F_{xi})$$

$$R_y = \sum(F_{yi})$$

Mål

$$R = \sqrt{R_x^2 + R_y^2}$$

Retning

$$\tan \varphi_R = \frac{R_y}{R_x}$$

Beliggenhet

$$a_R = \frac{\sum(F_i \cdot a_i)}{R}$$

Statisk bestemthet $U = L$

Antall likninger (generelt)

$$L = 3E + 2P + 0A$$

E = antall elementer (generelle)

P = antall partikler (knutepunkt)

A = antall aksialstaver

Antall ukjente reaksjonskrefter

$$U = O + 2P + 3F$$

O = Ytre opplagerkrefter (fastholdinger mot omgivelsene)

Fast innspenning = 3, Boltlager = 2, Glidelager/stanglager = 1

P = antall indre endeledd (knutepunkt / partikler)

F = antall indre faste innspenninger

Fagverk:

$$L = 2P$$

$$U = O + A$$

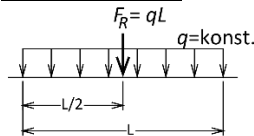
A = antall aksialstaver

Fordelt last (linjelast)

Resultantens mål:

$$F_R = \int_{x_1}^{x_2} q(x) dx$$

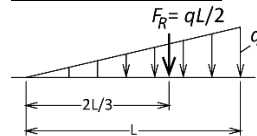
Jevnt fordelt last



Resultantens beliggenhet:

$$F_R \cdot x_R = \int_{x_1}^{x_2} q(x)x dx$$

Jevnt voksende last



Egentyngde

(bjelke)

$$q_W = \frac{G}{L} = g\rho A_{TS}$$

A_{TS} = Tverrsnittsareal

Hydrostatisk trykk

(platestripe)

$$q_H = g\rho hb$$

b = Lastbredde

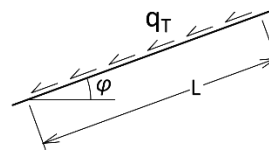
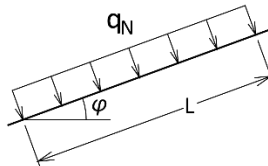
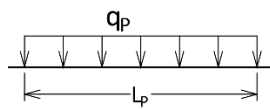
Projeksjonslast, q_P

Normalkomponent:

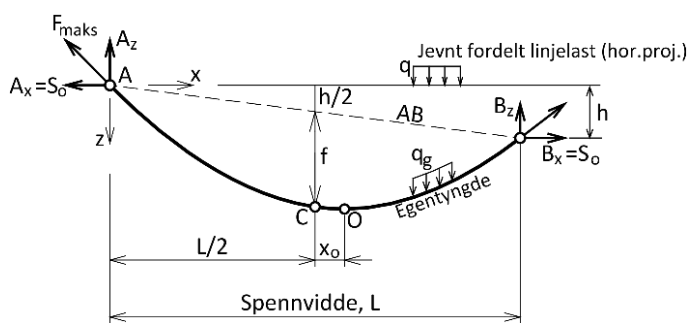
$$q_N = q_P \cos^2 \varphi$$

Tangentialkomponent:

$$q_T = q_P \cos \varphi \sin \varphi$$



Kabelformler



Kabelnedheng ved $L/2$:

(pilhøyde)

$$f = \frac{qL^2}{8S_0}$$

Projisert linjelast:

$$q = q_g \frac{s}{L}$$

Kabelparabel:

$$z(x) = \left(\frac{qL}{2S_0} + \frac{h}{L} \right) x - \frac{q}{2S_0} x^2 = \left(\frac{4f}{L} + \frac{h}{L} \right) x - \frac{4f}{L^2} x^2$$

Kabellengde:

$$s = L \left[1 + \frac{1}{2} \left(\frac{h}{L} \right)^2 + \frac{8}{3} \left(\frac{f}{L} \right)^2 \right]$$

Bunnpunkt, beligg. ift C:

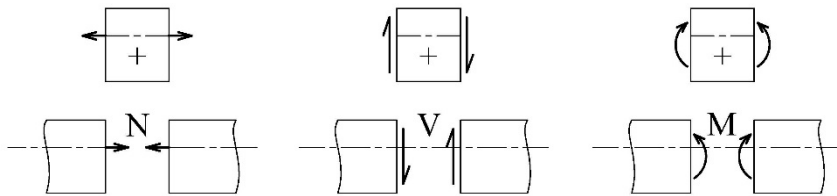
$$x_o = \frac{hL}{8f} \quad \text{og} \quad z_o = \frac{h^2}{16f}$$

Snittkrefter

$$\Delta V = - \int_{x_1}^{x_2} q(x) dx$$

$$\Delta M = \int_{x_1}^{x_2} V(x) dx$$

Fortegnsregler:



Fasthetslære, grunnleggende

Aksialspenninger:

$$\sigma_A = \frac{F}{A}$$

Tøyning:

$$\varepsilon = \frac{\Delta L}{L}$$

Materialegenskaper

Densitet, ρ

E-modul, E

Temp.utvidelseskoeff., α

Hookes lov:

$$\sigma = E\varepsilon$$

Termisk tøyning:

$$\varepsilon_T = \alpha \cdot \Delta T$$

Stål

7 850 kg/m³

210 GPa

11 · 10⁻⁶ m/m°C

Forlengelse av aksialbelastet stav:

$$\Delta L = \frac{FL}{EA}$$

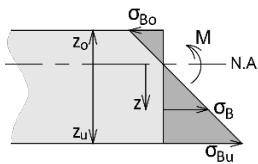
Aluminium

2 700 kg/m³

70 GPa

23,8 · 10⁻⁶ m/m°C

Bøyespenningsformelen



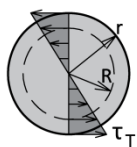
$$\sigma_B = \frac{M_B}{I_y} z$$

$$\sigma_{B,maks} = \frac{M_B}{W_y} \quad \text{hvor} \quad W_y = \frac{I_y}{|z_{maks}|}$$

Torsjonsskjærspenninger

Sirkulært tverrsnitt

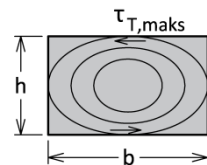
$$\tau_T = \frac{M_T R}{I_p}$$



Rektangulært tverrsnitt

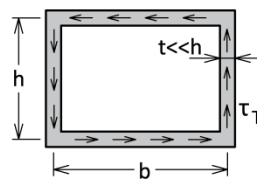
$b \geq h$

$$\tau_{T,maks} = \frac{M_T}{bh^2} \left[3 + 1,8 \frac{h}{b} \right]$$



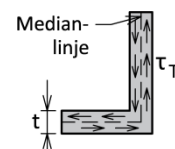
Tynnvegget rektangulær boks

$$\tau_T = \frac{M_T}{2bht}$$



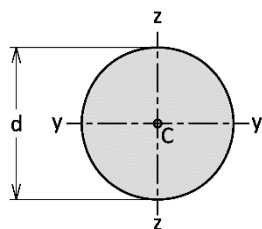
Tynnvegget åpent tverrsnitt

$$\tau_T = \frac{3M_T}{Lt^2}$$

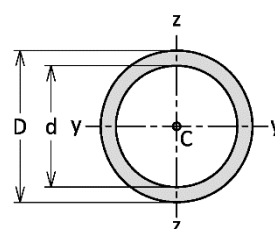


$L =$ medianlinjens lengde

Polart arealmoment



$$I_p = \frac{\pi}{32} d^4 = \frac{\pi}{2} r^4$$



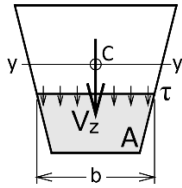
$$I_p = \frac{\pi}{32} (D^4 - d^4)$$

$$I_p \approx 2\pi t R^3$$

Skjærspenninger

Klipping: $\tau = \frac{V}{A_{TS}}$ $A_{TS} = \text{Tverrsnittsareal}$

Bjelketeori



$$\tau = \frac{V}{I_y b} S$$

$S = \text{arealmoment av A-flaten om y-aksen}$

Rektangulært tverrsnitt

$$\tau_{maks} = \frac{3V}{2A}$$

Sirkulært tverrsnitt

$$\tau_{maks} = \frac{4V}{3A}$$

Tynnvegget rør

$$\tau_{maks} = 2 \frac{V}{A}$$

Flytekriterier, $\sigma_F = \text{flytegrense}$

Mises-kriteriet: $\sigma_M < \sigma_F$

$\sigma + \tau$ $\sigma_M = \sqrt{\sigma^2 + 3\tau^2}$

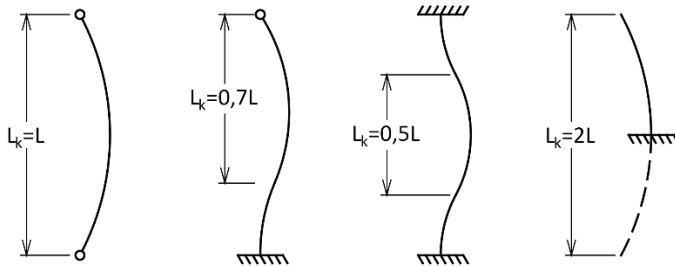
Plan spenning $\sigma_M = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{zx}^2}$

Generell spenningstilstand: $\sigma_M = \sqrt{(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x) + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$

Tresca-kriteriet: $\sigma_T = |\sigma_{maks} - \sigma_{min}| < \sigma_F$

Hovedspenninger: $\left. \begin{matrix} \sigma_1 \\ \sigma_2 \end{matrix} \right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$

Knekking



Elastisk knekking
(Eulerlast):

$$F_E = \frac{\pi^2 EI_o}{L_K^2}$$

Eulerspenning: $\sigma_E = \frac{\pi^2 E}{\lambda^2}$

Plastisk knekking
(Johnsons parabel):

$$\sigma_K = \sigma_F - \frac{1}{E} \left[\frac{\sigma_F \cdot \lambda}{2\pi} \right]^2 \quad \text{for } \epsilon < 20, \lambda_{1/2} >$$

Treghetsradius:

$$i_y = \sqrt{I_y/A}$$

Slankhet:

$$\lambda = \frac{L_K}{i}$$

Relativ slankhet:

$$\bar{\lambda} = \frac{\lambda}{\lambda_1}$$

Slankhet hvor $\sigma_E = \sigma_F$:

$$\lambda_1 = \pi \sqrt{\frac{E}{\sigma_F}}$$

Slankhet hvor
 $\sigma_E = \sigma_F/2$:

$$\lambda_{1/2} = \pi \sqrt{\frac{2E}{\sigma_F}}$$

Fluidstatikk

Tilstandslikning for ideell gass:

$$p = \rho RT$$

T = Temperatur angitt i Kelvin ($^{\circ}\text{C} + 273$)

$R = \frac{R_u}{M}$ = Gasskonstant for aktuell gass

$R_u = 8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}}$ = universell gasskonstant

M = Gassens molmasse (hentes fra materialtabeller)

Hydrostatisk trykk:

$$p = p_o + \rho gh$$

p_o = omgivelsestrykk

Resultantkraft på plan flate:

$$F = (p_o + \rho gh_c)A$$

A = Flatens areal

h_c = Dybde til flatens arealsenter

Trykksenter, plan flate:

$$h_p = h_c + \frac{I_c \sin^2 \varphi}{\left(h_c + \frac{p_o}{g\rho}\right)A}$$

φ = flatens vinkel ift. horisontalplan

Trykksenter, vertikal plan flate uten p_o .

$$h_p = h_c + \frac{I_c}{h_c A}$$

Trykktank med indre overtrykk

Tangentielle normalspenninger:

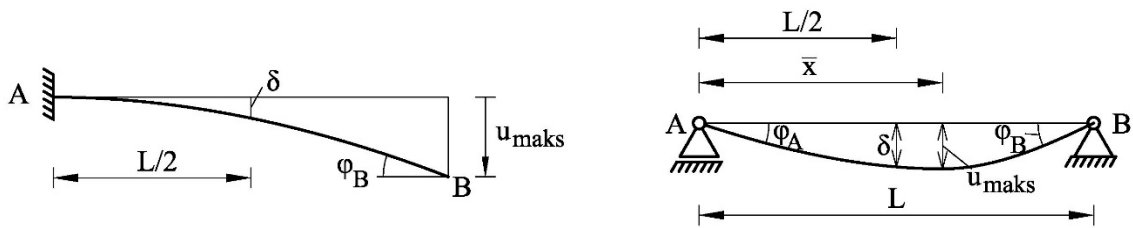
$$\sigma_{\varphi} = \frac{r}{t} p$$

Langsgående normalspenninger:

$$\sigma_L = \frac{r}{2t} p$$

Deformasjon av enkle bjelker

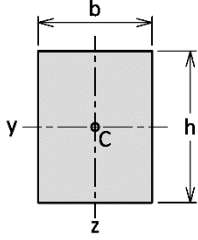
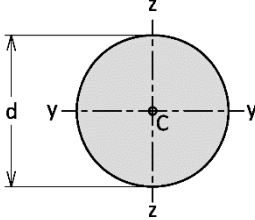
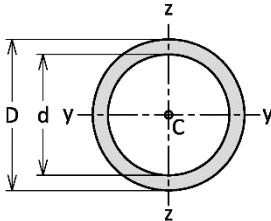
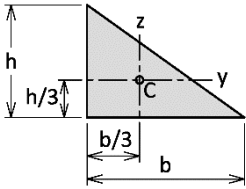
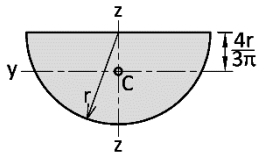
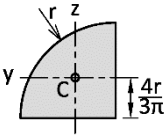
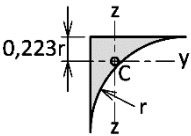
Største mulige utbøyning: $u_{maks} = \frac{L^2}{8EI} M_{maks}$



EI_y = bøystivhet, konstant
 δ = utbøyning på midten
 u_{maks} = største utbøyning
 u_{last} = utbøyning under punktlast
 \bar{x} = avstand fra A til punkt med maks. utbøyning
 φ_A og φ_B = tangenthelning ved hhv. A og B [rad.]

1		$u_{maks} = \frac{qL^4}{8EI}$ $\varphi_B = \frac{qL^3}{6EI}$ $\delta = \frac{17}{384} \frac{qL^4}{EI}$
2		$u_{maks} = \frac{FL^3}{3EI}$ $\varphi_B = \frac{FL^2}{2EI}$ $\delta = \frac{5}{48} \frac{FL^3}{EI}$ $u(x) = \frac{F}{EI} \left(\frac{1}{6} x^3 - \frac{L}{2} x^2 \right)$
3		$u_{maks} = \frac{ML^2}{2EI}$ $\varphi_B = \frac{ML}{EI}$
4		$u_{maks} = \frac{ML^2}{9\sqrt{3}EI}$ $\bar{x} = \left(1 - \frac{\sqrt{3}}{3} \right) L$ $\varphi_A = \frac{ML}{3EI}$ $\varphi_B = \frac{ML}{6EI}$ $\delta = \frac{ML^2}{16EI}$
5		$u_{maks} = \frac{Fb(L^2 - b^2)^{1.5}}{9\sqrt{3}EI}$ $\bar{x} = \sqrt{\frac{L^2 - b^2}{3}}$ $u_{last} = \frac{Fa^2b^2}{3LEI}$ $\delta = \frac{F \cdot b(3L^2 - 4b^2)}{48EI}$ $\varphi_A = \frac{F \cdot ab(L + b)}{6L \cdot EI}$ $\varphi_B = \frac{F \cdot ab(L + a)}{6L \cdot EI}$ $u(x) = \frac{Fbx}{6L \cdot EI} (L^2 - b^2 - x^2), \quad x \leq a$
6		$\delta = u_{maks} = \frac{FL^3}{48EI}$ $\varphi_A = \varphi_B = \frac{FL^2}{16EI}$
7		$\delta = u_{maks} = \frac{5}{384} \frac{qL^4}{EI}$ $\varphi_A = \varphi_B = \frac{qL^3}{24EI}$

Flategeometri

Snittflate m/arealsenter	Areal	Annet arealmoment
	$A = bh$	$I_y = \frac{bh^3}{12}$ $I_z = \frac{hb^3}{12}$
	$A = \frac{\pi}{4}d^2 = \pi r^2$	$I_y = I_z =$ $\frac{\pi}{64}d^4 = \frac{\pi}{4}r^4$
	$A = \frac{\pi}{4}(D^2 - d^2)$	$I_y = I_z =$ $\frac{\pi}{64}(D^4 - d^4)$ $I \approx \pi t R^3$
	$A = \frac{bh}{2}$	$I_y = \frac{bh^3}{36}$ $I_z = \frac{hb^3}{36}$
	$A = \frac{\pi}{2}r^2$	$I_y = 0,11r^4$ $I_z = \frac{\pi}{8}r^4$
	$A = \frac{\pi}{4}r^2$	$I_y = I_z =$ $\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)r^4 \approx$ $0,0549r^4$
 <p> $y_c = \frac{5 - 1,5\pi}{6 - 1,5\pi}r \approx 0,223r$ </p>	$A = \left(1 - \frac{\pi}{4}\right)r^2$	$I_y = I_z = 0,0075r^4$